OCR
RECOGNISING ACHIEVEMENT

Monday 23 January 2012 – Morning

AS GCE MATHEMATICS

4728 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4728
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Particles P and Q, of masses 0.3 kg and 0.5 kg respectively, are moving in the same direction along the same straight line on a smooth horizontal surface. P is moving with speed $2.2\,\mathrm{m\,s^{-1}}$ and Q is moving with speed $0.8\,\mathrm{m\,s^{-1}}$ immediately before they collide. In the collision, the speed of P is reduced by 50% and its direction of motion is unchanged.

(i) Calculate the speed of Q immediately after the collision. **[4]**

(ii) Find the distance PQ at the instant 3 seconds after the collision. **[2]**

2 In the sport of curling, a heavy stone is projected across a horizontal ice surface. One player projects a stone of weight 180 N, which moves 36 m in a straight line and comes to rest 24 s after the instant of projection. The only horizontal force acting on the stone after its projection is a constant frictional force between the stone and the ice.

(i) Calculate the deceleration of the stone. **[2]**

(ii) Find the magnitude of the frictional force acting on the stone, and calculate the coefficient of friction between the stone and the ice. **[4]**

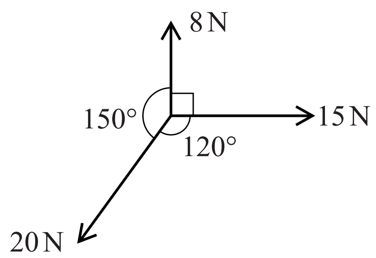
3 A car is travelling along a straight horizontal road with velocity $32.5\,\mathrm{m\,s^{-1}}$. The driver applies the brakes and the car decelerates at $(8 - 0.6t)\,\mathrm{m\,s^{-2}}$, where t s is the time which has elapsed since the brakes were first applied.

(i) Show that, while the car is decelerating, its velocity is $(32.5 - 8t + 0.3t^2)\,\mathrm{m\,s^{-1}}$. **[3]**

(ii) Find the time taken to bring the car to rest. **[2]**

(iii) Show that the distance travelled while the car is decelerating is 75 m. **[4]**

4



Three horizontal forces of magnitudes 8 N, 15 N and 20 N act at a point. The 8 N and 15 N forces are at right angles. The 20 N force makes an angle of 150° with the 8 N force and an angle of 120° with the 15 N force (see diagram).

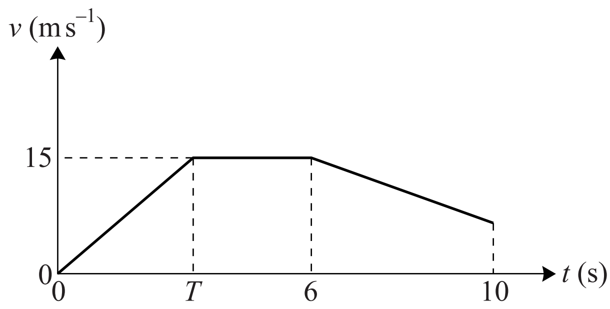
(i) Calculate the components of the resultant force in the directions of the 8 N and 15 N forces. **[3]**

(ii) Calculate the magnitude of the resultant force, and the angle it makes with the direction of the 8 N force. **[4]**

The directions in which the three horizontal forces act can be altered.

(iii) State the greatest and least possible magnitudes of the resultant force. **[2]**

5



The diagram shows the (t, v) graph of an athlete running in a straight line on a horizontal track in a 100 m race. He starts from rest and has constant acceleration until he reaches a speed of $15\,\mathrm{m\,s^{-1}}$ when $t = T$. He maintains this constant speed until he decelerates at a constant rate of $1.75\,\mathrm{m\,s^{-2}}$ for the final 4 s of the race. He completes the race in 10 s.

(i) Calculate T. **[5]**

The athlete races against a robot which has a displacement from the starting line of $(3t^2 - 0.2t^3)$ m, at time t s after the start of the race.

(ii) Show that the speed of the robot is $15\,\mathrm{m\,s^{-1}}$ when $t = 5$. **[3]**

(iii) Find the value of t for which the decelerations of the robot and the athlete are equal. **[3]**

(iv) Verify that the athlete and the robot reach the finish line simultaneously. **[2]**

6 A particle P of mass 0.3 kg is projected upwards along a line of greatest slope from the foot of a plane inclined at 30° to the horizontal. The initial speed of P is $4\,\mathrm{m\,s^{-1}}$ and the coefficient of friction is 0.15. The particle P comes to instantaneous rest before it reaches the top of the plane.

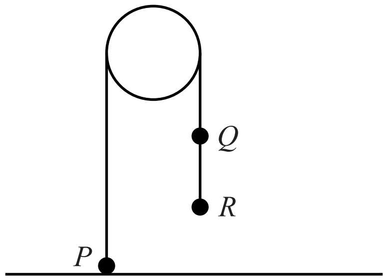
(i) Calculate the distance P moves up the plane. **[6]**

(ii) Find the time taken by P to return from its highest position on the plane to the foot of the plane. **[4]**

(iii) Calculate the change in the momentum of P between the instant that P leaves the foot of the plane and the instant that P returns to the foot of the plane. **[3]**

[Question 7 is printed overleaf.]

7



Particles P and Q, of masses m kg and 0.05 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth pulley. Q is attached to a particle R of mass 0.45 kg by a light inextensible string. The strings are taut, and the portions of the strings not in contact with the pulley are vertical. P is in contact with a horizontal surface when the particles are released from rest (see diagram). The tension in the string QR is 2.52 N during the descent of R.

(i) (a) Find the acceleration of R during its descent. **[2]**

(b) By considering the motion of Q, calculate the tension in the string PQ during the descent of R. **[3]**

(ii) Find the value of m. **[3]**

R strikes the surface 0.5 s after release and does not rebound. During their subsequent motion, P does not reach the pulley and Q does not reach the surface.

(iii) Calculate the greatest height of P above the surface. **[8]**

Question			Answer	Marks	Guidance
1	(i)		Total momentum before = $0.3 \times 2.2 + 0.5 \times 0.8$ Mom P after = $0.3 \times 2.2/2$ $0.3 \times 2.2 + 0.5 \times 0.8 = 0.3 \times 2.2/2 + 0.5v$ $v = 1.46$ ms^{-1}	B1 B1 M1 A1 **[4]**	Allow inclusion of g 0.33, accept $0.33g$ and negative term Allow $0.33g = 0.5gv - 0.5g \times 0.8$ M1 Allow from inclusion of g
1	(ii)		PQ = $3 \times 1.46 - 3 \times 2.2/2$ PQ = 1.08 m	M1 A1 **[2]**	$3(1.46 - 2.2/2)$ Accept $3 \times 1.46 - 2.2/2$
2	(i)		$36 = 0 +/- \ a24^2/2$ $a = +/- 0.125$ ms^{-2} OR $U = \pm 24a$ and $0^2 = (24a)^2 \pm 2a36$ $a = \pm 0.125\text{ms}^{-2} = \pm \frac{1}{8}\text{ms}^{-2}$	M1 A1 **[2]** M1 A1	$s = vt - at^2/2 = 0\ ^{+}/\text{-}\ at^2/2$ OR $s = ut\ ^{+}/\text{-}\ at^2/2$ $^1/_8$ Use both $0 = u \pm 24a$ *and* $0^2 = u^2 \pm 2a36$ $U = 3$ ms^{-1}
2	(ii)		$(180/g)a = Fr$ $Fr = \pm 2.3(0)$ N $\mu = 2.3/180$ $\mu = 0.0128$	M1 A1 M1 A1 **[4]**	Mass = 18.367…kg. Regard $180a = Fr$ as MR May be implied. $Fr = 22.5$ MR −1 Fr and R both +ve or both -ve, $\mu = 22.5/(180 \times 9.8)$ if MR Award if MR
3	(i)		$v = \pm\int -8 + 0.6t \ \mathrm{d}t$ $v = {}^{+}/\text{-}(-8t + 0.6t^2/2) \quad (+ c)$ $v = 32.5 - 8t + 0.3t^2$ AG	M1 A1 A1 **[3]**	Integrates accn or decn (Although only $v = -8t + 0.6t^2/2$ (+ c) is correct) ONLY FROM $v = \int -8 + 0.6t \ \mathrm{d}t$ OR $v = -\int 8 - 0.6t \ \mathrm{d}t$ and explicit $t = 0$, $v = 32.5$ so $c = 32.5$
3	(ii)		$0.3t^2 - 8t + 32.5 = 0$ $t = 5$	M1 A1 **[2]**	Starts to solve 3 term QE, either the given ans in (i) or the candidate's answer in (i) with v set = 0. Needs valid formula or factors which give 2 correct coefficients Accept as one of a pair only if the other value is 65/3 = 21.66....

Question			Answer	Marks	Guidance
3	(iii)		$s = \int 0.3t^2 - 8t + 32.5 \, dt$ $s = 0.3t^3/3 - 8t^2/2 + 32.5t\ (+ c)$ $D = 0.3 \times 5^3/3 - 8 \times 5^2/2 + 32.5 \times 5\ (+ c)$ $D = 75$ AG	M1 A1 M1 A1 **[4]**	Integrates an expression for velocity Accept omission of c Substitutes cv(smaller and +ve ans**(ii)**) or uses limits, $[\ ]_0^{smaller+vecv(ii)}$ Explicit evaluation needed. Accept + c
4	(i)		(X =)15 – 20cos60, 15 – 20sin30 *OR* (Y =)8 – 20cos30, 8 – 20sin60 (X =) 5 N (34.048.. if in rad mode) (Y =) –9.32 N (4.9149.. if in rad mode)	M1 A1 A1 **[3]**	Accept (X =) 15 + 20cos120, (Y =) 8 + 20cos 150, and R A =100° Must be +ve Must be –ve. Allow 8-10√3
4	(ii)		$R^2 = (+/-9.32)^2 + 5^2$ $R = 10.6$ N $\tan\theta = (+/-9.32)/5$ Angle = 152°	M1 A1 ft M1 A1 **[4]**	Uses Pythagoras on ans**(i)**, neither component 8 or 15 $\sqrt{(X(\mathbf{i})^2 + Y(\mathbf{i})^2)}$ Finds any relevant angle with 8 N or 15 N, neither component 8 or 15 CAO, must be 3sf or better
4	(iii)		(Greatest =) 43 N (Least =) 0 N	B1 B1 **[2]**	
5	(i)		$S_{dec} = 15 \times 4 - 1.75 \times 4^2/2$ $S_{dec} = 46$ $100 - 46 = 15T/2 + 15(10 - 4 - T)$ $(= 15 \times 6 - 15T/2)$ $54 = 90 - 7.5T$ $T = 4.8$	M1 A1 M1 A1ft A1 **[5]**	Or $v = 15 - 1.75 \times 4$ and $s = (15 + v)/2 \times 4$ May be implied Any attempt at combined 3 stage distances being 100 Simplification not essential. ft cv(S_{dec}**(i),** numerical)
5	(ii)		$V_R = d(3t^2 - 0.2t^3)/dt$ $V_R = 6t - 0.6t^2$ $V_R(5)\ (= 6 \times 5 - 0.6 \times 5^2) = 15$ ms^{-1} AG	M1 A1 A1 **[3]**	Attempt at differentiating S_R Accept $V_R = 2 \times 3t - 3 \times 0.2t^2$ Must show explicit substitution

Question			Answer	Marks	Guidance
5	(iii)		$A_R = d(6t - 0.6t^2)/dt$ $6 - 1.2t = -1.75$ $t = 6.46$	M1* D*M1 A1 **[3]**	Attempt at differentiating V_R Must be –1.75 or $1.2t - 6 = 1.75$ (i.e. employs <u>deceleration</u>)
5	(iv)		$S_R(10) = 3 \times 10^2 - 0.2 \times 10^3$ $S_R(10) = 100$ *OR* $3t^2 - 0.2t^3 = 100$ $t = 10$ which is how long the athlete takes to finish	M1 A1 **[2]** M1 A1	Substitutes 10 into S_R formula Sets up and tries to solve equation for robot Needs comment about athlete or both finishing race in 10 s
6	(i)		$R = 0.3g\cos30$ $Fr = 0.15 \times 0.3g\cos30$ $0.3a = -0.3g\sin30 - 0.15 \times 0.3g\cos30$ $a = -6.17$ $0 = 4^2 - 2 \times 6.17s$ $s = 1.3(0)$ m	B1 M1 M1 A1 M1 A1ft **[6]**	$R = 2.546$ N. May be shown on diagram $0.15 \times$ cv(R), $Fr = 0.382$ N2L, two forces inc. 0.3gCorS30 and friction Accept positive value Using a from above ft(8/ |cv(a)|) CorS30 means cos30 or sin30
6	(ii)		$0.3a = 0.3g\sin30 - 0.382$ $a = 3.63$ $1.3 = 3.63t^2/2$ $t = 0.845$ s	M1 A1 M1 A1 **[4]**	N2L, diff. of two forces inc. 0.3gCorS30 and friction Using cv(s(**i**)), and a not a(**i**) nor 9.8 Rounds to 0.85 if 2 sig fig. CorS30 means cos30 or sin30
6	(iii)		$V = 3.63 \times 0.845$ *OR* $V = \sqrt{(2 \times 3.63 \times 1.3)}$ *OR* $V = 2 \times 1.3/0.845$ ($V = 3.07$) Mom change = +/-(0.3 × 4 + 0.3 × 3.07) Mom change = +/-2.12 kgms^{-1}	M1 M1 A1 **[3]**	cv(a(**ii**) × t(**ii**)) *OR* cv($\sqrt{2}$×a(**ii**)×s(**i**) *OR* cv(2×s(**i**)/ t(**ii**)), a(**ii**) not a(**i**) nor 9.8 +/-(0.3 × 4 +/– 0.3 × speed(return)), 0<speed(return) < 4, g omitted

Question			Answer	Marks	Guidance
7	(i)	(a)	$0.45a = 0.45g - 2.52$ $a = 4.2 \text{ ms}^{-2}$	M1 A1 **[2]**	N2L for R. 2 vertical forces. Accept +/-$0.45a = 0.45g$ +/– 2.52 Accept –4.2
7	(i)	(b)	$0.05 \times 4.2 = 0.05g + 2.52 - T$ $T = 0.05 \times 9.8 + 2.52 - 0.05 \times 4.2$ $T = 2.8$ N	M1 A1 ft A1 **[3]**	N2L for Q, 3 vertical forces, $0.05 \times 4.2 = 0.05g$ +/- 2.52 +/– T accn not 9.8; 0.5g is TWO vertical forces ($0.45g + 0.05g$) not MR ft cv(a(**i**)). Any equivalent form of equation *ACCEPT A COMBINED Q AND R METHOD* $(0.45 + 0.05) \times 4.2 = 0.45g + 0.05g$ +/–T M1 $(0.45 + 0.05) \times 4.2 = 0.45g + 0.05g - T$ A1ft $T = 2.8$ N A1
7	(ii)		$\pm 4.2m = T - mg$ *OR* $\pm 4.2 = (0.05g + 0.45g - mg)/(0.05 + 0.45 + m)$ $4.2m = 2.8 - mg$ *OR* $9.8m + 4.2m = 2.8$ $m = 0.2$	M1 A1 ft A1 **[3]**	N2L for P, difference of 2 vertical forces, accn cv(a(**i**)) ±cv(a(**i**)) = (wt P + wt Q – wt R) / sum of masses ft cv(T(**ib**)) Any equivalent form of equation with cv(a(i))
7	(iii)		*BEFORE R STRIKES SURFACE* $v = 4.2 \times 0.5$ $v = 2.1$ $s = 2.1^2/(2 \times 4.2) = 4.2 \times 0.5^2/2$ *AFTER R STRIKES SURFACE* +/-$0.2a = T - 0.2g$ *OR* +/-$0.05a = 0.05g - T$ +/-$0.2a = T - 0.2g$ *AND* +/-$0.05a = 0.05g - T$ a = +/–5.88 $S = 2.1^2 / (2 \times 5.88)$ *TOTAL JOURNEY* Distance = (0.375 + 0.525) = 0.9m	 M1* A1 M1 M1 A1 A1 D*M1 A1 **[8]**	 Find Speed when R hits surface, using a(**i**) Distance R falls (0.525 m). Accept +/-$4.2 \times 0.5^2/2$ N2L for either P (with cv(m)) or Q Correct equations for both P and Q *OR* combination $0.05g\ (-T + T) - 0.2g$ = +/–$(0.2a + 0.05a)$ M1A1 Distance P rises after R hits ground (0.375), a not a(**i**) or 9.8